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Effects of quantum space time foam in the neutrino sector

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Abstract. We discuss violations of CPT and quantum mechanics due to interactions of neutrinos with space-time quantum foam. Neutrinoless double beta decay and oscillations of neutrinos from astrophysical sources (supernovae, active galactic nuclei) are analysed. It is found that the propagation distance is the crucial quantity entering any bounds on EHNS parameters. Thus, while the bounds from neutrinoless double beta decay are not significant, the data of the supernova 1987a imply a bound being several orders of magnitude more stringent than the ones known from the literature. Even more stringent limits may be obtained from the investigation of neutrino oscillations from active galactic nuclei sources, which have an impressive potential for the search of quantum foam interactions in the neutrino sector.

PACS. 04.60.-m Quantum gravity – 14.60.Pq Neutrino mass and mixing – 23.40.-s β decay; double β decay; electron and muon capture – 95.85.Ry Neutrino, muon, pion, and other elementary particles; cosmic rays

1 Introduction

While in the context of local quantum field theories CPT has to be conserved, CPT-violating effects may show up in the framework of quantum gravity. As an example, Hawking radiation of black holes can be understood as a pair creation process near the event horizon, with one particle falling into the black hole and the other one escaping. Since with the particle falling into the black hole some phase information of the quantum state is lost, the thermic final state is a mixed state rather than a pure one. As Hawking has pointed out [1], such an evolution of a pure state into a mixed state violates the laws of conventional quantum mechanics (QMV). If the space time possesses a foamy structure at the Planck scale, including the creation and annihilation of black holes with Planck radius and Planck lifetime, such effects also may influence microscopical processes in the vacuum [2]. In the following Page [3] showed that such processes violate also CPT and the possibility of experimental tests in the K_0-$\bar{K}_0$ sector was discussed by Eberhard [4]. Ellis, Hagellin, Nanopoulos and Srednicki independently developed an evolution equation formalism in the space of density matrices [5] containing three CPT-violating (EHNS) parameters α, β, γ which have a dimension of mass and which might be expected to be of order $m_{\mathrm{K}}^2/M_{\mathrm{Pl}} \sim 10^{-20}$ GeV in the kaon sector. Recently the topic has been reconsidered by Ellis, Mavromatos and Nanopoulos [6] and Huet and Peskin [7]. CPT-violating processes in the neutrino sector have been discussed for the first time by Liu *et al.* [8] and in the following in [9], where neutrino oscillations due to CPT violation has been discussed as a solution to the solar neutrino problem. Recently another paper [10] explored the possibility of explaining the atmospheric neutrino anomaly with quantum foam effects and came to a negative conclusion. In this note we extend the discussion of quantum foam effects in the neutrino sector to the cases of neutrinoless double-beta decay and oscillations of neutrinos from astrophysical sources, supernovae as well as active galactic nuclei. New, extremely stringent bounds are found improving constraints found in the literature by several orders of magnitude.

2 Density matrix formalism

For mixed states it is useful to work in the framework of the density matrix formalism, following the methodology as presented in ref. [8]. We start with the Schrödinger equation for the density matrix,

$$i\frac{\mathrm{d}}{\mathrm{d}t}\rho = [H, \rho]. \tag{1}$$

Here ρ is the density matrix of the system, which can be expanded in the Pauli matrix basis,

$$\rho = \rho^0 I + \rho^i \sigma^i, \tag{2}$$

where I is the unity matrix and σ^i are the Pauli matrices. In [9] a lepton-number-violating parametrization for

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the evolution equation of the components of the density matrix has been assumed:

$$\frac{d}{dt}\begin{pmatrix}\rho^0\\ \rho^1\\ \rho^2\\ \rho^3\end{pmatrix} = 2\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & \Delta m^2/(4E) & 0\\ 0 & -\Delta m^2/(4E) & -\alpha & -\beta\\ 0 & 0 & -\beta & -\gamma\end{pmatrix}\begin{pmatrix}\rho^0\\ \rho^1\\ \rho^2\\ \rho^3\end{pmatrix}. \quad (3)$$

Here $\beta \ll \alpha, \gamma$ [5,7,8]. In [8,9] also an alternative, lepton-number-conserving parametrization has been discussed. However, this parametrization will not influence neither the double-beta decay observable nor the oscillation probability in the asymptotics of large propagation distances compared to the standard case of neutrino masses [9]. Thus we will concentrate on the lepton-number violating case in the following. It should be mentioned however that a full analysis of the generalized dynamics requires six parameters [11].

Moreover, it should be stressed that this non-relativistic ansatz may not be suitable to describe ultrarelativisic particles such as neutrinos. However, while the covariant treatment of open quantum systems is still an unsolved problem, the density matrix ansatz has been successfully used in previous works to derive the "standard" mass mechanism neutrino oscillation probability also, see [10]. Thus, while future works should improve the present ansatz, this approach seems to be suitable to provide at least a possibility for the comparison of the sensitivity of different experiments and a rough estimation for the order of magnitude of the obtained bounds.

3 Neutrinoless double-beta decay

Neutrinoless double-beta decay is one of the most sensitive tools in neutrino physics. It corresponds to two single-beta decays occuring simultaneous in one nucleus, with a virtual neutrino propagating between the vertices. Important impact of this process has been derived on the reconstruction of the neutrino mass spectrum, physics beyond the standard model as well as more exotic phenomena such as violations of the equivalence principle or Lorentz invariance (for an overview see [12,13]). In the following we will study the potential of neutrinoless double-beta decay for searches for CPT violations due to quantum foam interactions in the neutrino sector. The observable measured in neutrinoless double-beta decay is the ee entry of the neutrino mass matrix in the flavor space,

$$m_{ee} = \bar{m} - \frac{\delta m}{2}\cos(2\theta) \quad (4)$$

in a two-neutrino scenario with $\bar{m} = (m_1 + m_2)/2$ and $\delta m = (m_2 - m_1)$ and $m_{1,2}$ being the mass eigenstates. This quantity will be modified in the presence of QMV. The recent experimental constraint is $m_{ee} < 0.3$ eV, obtained from the Heidelberg-Moscow experiment searching for double-beta decays of ^{76}Ge [14]. The GENIUS project will be sensitive to $m_{ee} = 10^{-2} - 10^{-3}$ eV [15]. In the density matrix formalism the double-beta decay observable can be expressed as follows:

$$\mathrm{Tr}(\rho_{\nu_e}\mathcal{O}) = \mathrm{Tr}\begin{pmatrix}\rho_0+\rho_3 & \rho_1 - i\rho_2\\ \rho_1 + i\rho_2 & \rho_0+\rho_3\end{pmatrix}\cdot\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix}$$
$$= (m_1+m_2)\rho_0 + (m_2 - m_1)\rho_3. \quad (5)$$

The propagation time of the neutrino

$$t = \frac{1}{4\pi\Delta E} \simeq 6\cdot 10^{-24}\mathrm{s} \quad (6)$$

can be estimated by taking its energy to be of the size of the nuclear Fermi momentum $p_F \simeq 100$ MeV for ^{76}Ge. Assuming $\beta \ll \alpha, \gamma$, eq. (5) yields

$$\frac{d}{dt}\rho_0 = 0\,, \quad (7)$$
$$\frac{d}{dt}\rho_3 = -2\gamma\rho_3 \quad (8)$$

and thus, using eq. (4)

$$\rho_0 = \frac{1}{2} \quad (9)$$
$$\rho_3 = e^{-2\gamma t}\frac{\cos(2\theta)}{2}. \quad (10)$$

This implies

$$m_{ee}^{QMV} = \bar{m} + e^{-2\gamma t}\frac{\Delta m}{2}\cos 2\theta. \quad (11)$$

Due to the tiny propagation time (6) no significant variation of the double-beta decay observable is obtained. However, from this analysis we realize that the distance plays a crucial role in constraining the QMV parameters, so we shall consider the bounds on the neutrino oscillation probability where neutrinos are propagating over large distances.

4 Oscillations of neutrinos from astrophysical sources

In the following we study the effect of quantum mechanics violation in neutrino oscillations from astrophysical sources. The most distant sources that have been discussed in the context of neutrino oscillations are supernovae (SN) and active galactic nuclei (AGN). While astrophysical sources have been discussed in the context of QMV effects on life time measurements [16,17], they have not been considered for the case of QMV induced neutrino oscillations so far.

For the neutrino oscillation case we get the survival and disappearance oscillation probabilities [8,9]

$$P(\nu_x \to \nu_x) = \mathrm{Tr}[\rho_{\nu_x}(t)\rho_{\nu_x}], \quad (12)$$
$$P(\nu_x \to \nu_{x'}) = \mathrm{Tr}[\rho_{\nu_x}(t)\rho_{\nu_{x'}}], \quad (13)$$

respectively. Here the density matrices can be parametrized as

$$\rho_{\nu_x} = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}, \quad (14)$$

$$\rho_{\nu_{x'}} = \begin{pmatrix} \sin^2\theta & -\cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{pmatrix}. \quad (15)$$

As initial condition we assume

$$\rho(t=0) = \rho(\nu_e) \quad (16)$$

and thus [8,9]

$$\rho_0 = \frac{1}{2}, \quad (17)$$

$$\rho_1 = \frac{1}{2}\sin(2\theta), \quad (18)$$

$$\rho_2 = 0, \quad (19)$$

$$\rho_3 = \frac{1}{2}\cos(2\theta). \quad (20)$$

The interesting observable is the oscillation propability

$$P^{\rm QMV}_{\nu_x\to\nu_{x'}} = {\rm Tr}[\rho(t)\rho_x] = \frac{1}{2} - \frac{1}{2}e^{-\gamma L}\cos^2 2\theta - \frac{1}{2}e^{-\alpha L}\sin^2 2\theta\cos(\frac{\Delta m^2}{2E_\nu}L), \quad (21)$$

where $\beta \ll \alpha, \gamma$ has been assumed. For the n-flavour case the oscillation probability for large propagation distances is given by [9]

$$P^{\rm QMV}_{\nu_x\to\nu_{x'}} = \frac{1}{n} - \frac{1}{n}e^{-\gamma L}, \quad (22)$$

where L is the propagation distance of the neutrinos.

This QMV oscillation probability can easily be distinguished from the asymptotics of the "standard" mass induced oscillation probability:

$$P^{\rm mass}_{\nu_x\to\nu_{x'}} = \frac{\sin^2 2\theta}{2}. \quad (23)$$

The quantity $P^{\rm mass}$ is fixed experimentally to $P^{\rm mass}_{\nu_\mu\to\nu_\tau} \simeq 0.5$ due to the maximal mixing in atmospheric neutrinos [18] and $P^{\rm mass}_{\nu_e\to\nu_\tau} \lesssim 0.05$ due to the CHOOZ bound [19].

Supernovae 1987a: In supernovae strong neutrino oscillations will significantly distort the ν_e spectra at the earth, since the ν_e will aquire the spectra of the more energetic ν_μ and ν_τ. The distance is very large. As a result, the condition that QMV should satisfy the bound on the oscillation probability gives a very strong bound. In the case of supernova 1987a, $L \sim 50{\rm Mpc} \sim 7\cdot 10^{39}{\rm GeV}$, so that the observed constraint on the oscillation probability [20] $P^{\rm exp}_{\nu_e\to\nu_{\mu,\tau}} < 0.2$ is satisfied for the three-neutrino case when

$$\gamma < \frac{0.6}{L} \sim 10^{-40}{\rm GeV}. \quad (24)$$

We assumed here that $P^{\rm exp}$ is the accuracy with which the deviations from the asymptotics $1/n = 1/3$ can be measured. Due to the unknown energy dependence of the EHNS parameters and the Lorentz non-invariant ansatz it is difficult to compare these bounds with the bound coming from K-physics. Following [8] we assume γ to be of the order E_ν^2/M_{Pl} and scale the obtained bound by the neutrino energy to the kaon mass squared,

$$\gamma_\nu \propto \frac{E_\nu^2}{m_{\rm K}^2}, \quad (25)$$

implying $\gamma_{\rm K} < 10^{-37}{\rm GeV}$, which is an improvement of about 16 orders of magnitude. This disfavors strongly any solution of the solar or atmospheric neutrino problem by lepton-number-violating QMV effects. If one assumes that the same QMV parametrization is valid for the K-system, then any observational possibility in the K-system will also be excluded by the present constraints from the supernovae analysis. A relativistic treatment of the problem can modify this bound to some extent, but it is most unlikely that the modification is by several orders of magnitude.

Active Galactic Nuclei (AGN) : AGN can be intense sources of high energy neutrinos ($E_\nu \sim \mathcal{O}$ (1 PeV)) [21]. According to representative models the flux of these neutrinos is flavor dependent and the ν_τ flux is reduced by at least two orders of magnitude compared to the ν_e, ν_μ fluxes. A unique appearance signal of high energy ν_τ neutrinos can be a double-bang signal of the produced τ leptons: The first bang originates from the CC interaction of the τ neutrino and the second one from the hadronic decay of the τ lepton. Deep underwater or ice neutrino detectors have been estimated to be sensitive on neutrino oscillation probabilities of [21,22]

$$P^{\rm exp}_{\nu_{e,\mu}\to\nu_\tau} < 5\times 10^{-3}. \quad (26)$$

Since the QMV effects become strong for large distances and higher energies, it is likely that when we have data from the active galactic nuclei on neutrino oscillations, these bounds will be modified by several orders of magnitude. Considering the distance to be $L \sim 100{\rm Mpc}$ and the average energy of the neutrinos to be around 1 PeV, a bound on the neutrino oscillation of $P_{\nu_e\to\nu_\mu} < 5\times 10^{-3}$ will imply a corresponding bound on the QMV parameter

$$\gamma_\nu < 10^{-42}{\rm GeV}. \quad (27)$$

Translation to the kaon mass scale yields

$$\gamma_{\rm K} < 10^{-55}{\rm GeV}, \quad (28)$$

which would imply the by far strongest bound on QMV parameters. This will provide a decisive test for any contribution of lepton-number-violating QMV effects in the neutrino sector.

5 Conclusions

We studied the effects of violation of quantum mechanics due to quantum space time foam interactions in neutrino experiments. While the non-observation of neutrinoless double-beta decay does not give any significant constraint, the supernova 1987a implies a constraint being 16 orders of magnitude more stringent than the bounds known from the literature. This disfavors strongly any possibility of observable effects of lepton-number-violating QMV in any other experiments. The non-observation of QMV induced neutrino oscillations from active galactic nuclei will be able to improve this bound by many orders of magnitude. While the chosen non-relativistic ansatz might not be totally suitable for neutrinos, it should be at least useful to compare the sensitivity of different neutrino sources. Moreover the bounds obtained are that stringent, that, even in view of this ambiguity, they should be considered as the most restrictive ones.

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